Quantum Effects and Gravity

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Source of the gravitational field:
stress energy tensor of matter,
including the energy density

Classical physics: energy density is positive and
gravity is attractive

Quantum field theory: energy density can be
negative - possibility of repulsive gravity
Negative energy as a subvacuum effect—suppression of usual vacuum fluctuations

Example: Casimir effect

Perfectly reflecting plates: constant density negative energy

Casimir effect stress tensor: Brown & Maclay, DeWitt

\[
\langle T_{\mu\nu} \rangle = \rho \text{ diag}(1, -1, -1, 3)
\]

\[
\rho = -\frac{\hbar c \pi^2}{720 L^4}
\]

Gravity determines what is zero energy.
Effects of finite reflectivity - can there be a large positive self energy? (Helfer & Lang)

Classical electrostatics: attractive forces but positive energy density

Result: energy density can be negative, but requires high reflectivity or large separations (Sopova & LF)

\[ \omega_p = \text{plasma frequency} \]
Non-classical quantum states

An example: A superposition of the vacuum and a two photon state

\[ |\psi\rangle = \frac{1}{\sqrt{1 + \varepsilon^2}} (|0\rangle + \varepsilon|2\rangle) \quad |\varepsilon| \ll 1 \]

\[ \rho = \langle \psi | : T_{tt} : |\psi\rangle = 2\varepsilon \text{Re}(\langle 0 | : T_{tt} : |\psi\rangle) + O(\varepsilon^2) \]

can be negative at a given spacetime point
Negative energy density in a squeezed vacuum state:

Can be made arbitrarily negative at a given point by increasing the frequency of the mode

\[ \rho \propto \omega \sinh r \left[ \sinh r - \cosh r \cos(2\omega t) \right] \]

Another example:

Can be made arbitrarily negative at a given point by increasing the frequency of the mode
Possible effects of negative energy:

Repulsive gravity

Singularity avoidance

Violation of the weak energy conditions allows the singularity theorems to be evaded.

Traversable wormholes

Morris & Thorne

Negative energy is needed at the throat of the wormhole to cause light rays to defocus.
Faster than light travel - Alcubierre warp drive

Time Machines

Can modify a traversable wormhole or warp drive to travel backwards in time

Hawking’s Theorem: Negative energy is essential to build a time machine.
Violations of the second law of thermodynamics

Shine negative energy on a black hole and reduce its horizon area.
Violations of cosmic censorship: shine negative energy on an extreme black hole
Constraints on negative energy in Minkowski spacetime:

1) Total energy is non-negative - positivity of the Hamiltonian

2) Averaged weak energy condition -

\[ \int \langle T_{\mu\nu} \rangle \, u^\mu \, u^\nu \, d\tau \geq 0 \]

\( u^\mu \) = four velocity of an inertial observer

Neither of these conditions is strong enough to avoid negative energy problems; the positive energy could be very far from the negative energy.
3) Quantum inequalities -

\[ \int \langle T_{\mu\nu} \rangle \, u^\mu \, u^\nu \, g(\tau, \tau_0) \, d\tau \geq - \frac{C}{\tau_0^d} \]

\( g(\tau, \tau_0) \) = sampling function

\( \tau_0 \) = sampling time

\( C \) = positive constant

\( d \) = spacetime dimension
Some Minkowski space examples with a Lorentzian sampling function:

Two dimensions (1+1)

\[
\frac{\tau_0}{\pi} \int_{-\infty}^{\infty} \frac{\langle T_{\mu\nu} \rangle u^\mu u^\nu}{\tau^2 + \tau_0^2} \, d\tau \geq -\frac{1}{48\pi \tau_0^2}
\]

(Flanagan) Optimum bound

Four dimensions (3+1)

\[
\frac{\tau_0}{\pi} \int_{-\infty}^{\infty} \frac{\langle T_{\mu\nu} \rangle u^\mu u^\nu}{\tau^2 + \tau_0^2} \, d\tau \geq -\frac{3}{32\pi^2 \tau_0^4}
\]

(LF&Roman, Fewster&Eveson)
In the limit that $\tau_0 \to \infty$, we recover the averaged weak energy condition as a special case.

$$\int \langle T_{\mu\nu} \rangle u^\mu u^\nu \, d\tau \geq 0$$

A negative energy density cannot last longer than about

$$\Delta t = |\rho|^{-1/4}$$

(4D)
Physical implication:

The amount of negative energy that can be absorbed by a system in time $t$ is less than $1/t$.

In 4D, need the collecting area $< 1/t^2$.

This is less than the quantum energy uncertainty on this time scale.
We should not expect large macroscopic effects from negative energy.

No macroscopically observable violations of the second law or of cosmic censorship

Severe constraints on the parameters of wormholes and warp drives

\[ \ell = \text{local radius of curvature} \]

Einstein’s equations:

\[ \frac{1}{\ell^2} \approx \ell_P^2 \frac{1}{\ell^4} \]

\[ \Rightarrow \ell \approx \ell_P = \text{Planck length} \]

Pfenning, Roman, LF
Negative energy or related subvacuum effects might still be observable.

Some proposed laboratory experiments:

1) Effects on magnetic moments of spin systems

   LF, Grove & Ottewill

Basic idea: vacuum fluctuations cause de-alignment of spins and their suppression can cause a temporary re-alignment, or increase in magnetization.

   Very small effect.
2) Effects on atomic decay rates

LF & Roman

3) Effects on the speed of light in a nonlinear material

DeLorenci & LF
Why are quantum gravity effects so small?

Because the Planck length $\ell_P$ is very small compared to the scale of our experiments, $L$.

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} \text{ m}$$

Expect fractional effects of the order of a power of $\ell_P/L$.
An example of a quantum gravity effect: lightcone fluctuations.

Effects on light rays in classical gravity:

- Light deflection
- Shapiro time delay
- Light slowed by the sun’s gravitational field
Effects of quantum fluctuations of gravity:

angular blurring of images

variations in the flight times of light pulses

Some pulses (A) travel slower compared to flat space, and others (B) travel faster.
Ways to evade the smallness of the Planck length

1) Build an analog model in a non-gravitational system

2) Find an amplification mechanism
Non-linear optical materials and analog models of lightcone fluctuations

with Carlos Bessa, Vitorio DeLorenci, G. Menezes and Nami Svaiter

Basic Idea:

A background electric field in a non-linear material changes the effective lightcone for a probe pulse. This is analogous to the effect of a classical gravitational field on light.

If the background field has quantum fluctuations, then the effective lightcone for the probe pulse also fluctuates.

Result: \[
\frac{\Delta t}{t} \approx 10^{-9}
\] Measureable?
Enhancement of quantum gravity effects through secular growth

Formation of large scale structure from quantum fluctuations during inflation

Hawking effect

Non-cancellation of anti-correlated fluctuations
Summary

1) Quantum matter field can give rise to negative energy density and hence to repulsive gravity.

2) This can give rise to several exotic effects, such as wormholes and faster than light signals.

3) Quantum inequalities place limits on such effects, but small subvacuum effects might be observable.

4) Quantum gravity predicts fluctuations of the speed of light - light cone fluctuations.

5) This is a very small effect, but might be observable in an analog model.